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# Optimisation Techniques in Engineering: A Mathematical Perspective

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**ABSTRACT**

***This literature review provides a comprehensive examination of optimization techniques in engineering from a mathematical perspective, highlighting their foundational theories, classifications, multi-objective approaches, constraint handling methods, and hybrid techniques. The rapid advancement and adoption of meta-heuristic algorithms, particularly those inspired by biological and natural phenomena, underscore their effectiveness in solving complex engineering problems. The review discusses the mathematical underpinnings that facilitate the implementation of various optimization strategies, including multiparametric programming and differential geometry, which enhance problem-solving capabilities under uncertainty. Additionally, it explores the evolution of nature-inspired algorithms, including the Hippopotamus Optimization and Wild Horse Optimizer, alongside physics-inspired and human-inspired methods that showcase significant improvements in performance through hybridization and enhancement strategies. Multi-objective optimization approaches and constraint handling techniques are analyzed for their roles in addressing real-world challenges, with notable advancements in methods like MOEDO and NSGA-II. Applications across diverse engineering domains demonstrate the practical benefits of these optimization techniques, yielding improvements in efficiency and sustainability. The review concludes with an acknowledgment of ongoing challenges and future research directions, emphasizing the need for refined performance metrics and structured frameworks for algorithm selection. As optimization techniques continue to evolve, their synergy with artificial intelligence and machine learning is poised to drive further innovations in engineering solutions.***

## 1. Introduction

Optimization techniques have become indispensable tools in modern engineering practice, enabling engineers to find optimal solutions to complex problems. Engineering optimization has gained significant attention from research teams globally, becoming an integral component of contemporary mathematical modelling and control of processes and systems [1]. The rapid pace of industrialization has intensified the need for effective optimization solutions, leading to the widespread adoption of meta-heuristic algorithms [2]. From a historical perspective, the repertoire of optimization algorithms has expanded dramatically, with over 300 new methodologies developed in the last decade alone, out of more than 600 total techniques [2]. This explosion of new approaches underscores the critical importance of developing a sophisticated understanding of these novel methods.

The mathematical perspective on optimization techniques provides the theoretical foundation necessary for their effective implementation in engineering contexts. Most real-world problems in engineering involve some type of optimization problems that are often constrained [3]. Researchers have extensively investigated various techniques to address constrained single-objective and multi-objective evolutionary optimization across numerous fields, spanning both theoretical and applied domains [3]. Optimization algorithms are designed to find solutions to various problems and determine optimal outcomes with high accuracy and low error, improving performance across diverse fields including machine learning, operations research, physics, chemistry, and engineering [4].

In recent years, there has been a paradigm shift in the application of biological and natural phenomena to inform meta-heuristic optimization strategies. This trend reflects an increasing recognition of the effectiveness of bio-inspired methodologies in addressing intricate engineering problems, providing solutions that exhibit rapid convergence rates and unmatched fitness scores [2]. As technology continues to advance, optimization algorithms are increasingly needed to address complex real-world challenges and drive innovation across all disciplines, with quantitative leaps achieved in improving their efficiency through diverse information sources tailored to specific optimization problems, based on scientific and organized foundations [4].

This literature review aims to provide a comprehensive examination of optimization techniques in engineering from a mathematical perspective, exploring mathematical foundations, algorithm classifications, multi-objective approaches, constraint handling methods, hybrid techniques, engineering applications, performance evaluation methodologies, and future research directions.

## 2. Mathematical Foundations of Optimization in Engineering

The mathematical foundations of optimization provide the theoretical underpinnings for algorithm development and implementation in engineering contexts. Multiparametric programming represents a strategy that offers a holistic perspective for solving mathematical programming problems under uncertainty. This approach enables the derivation of the optimal solution as a function of uncertain parameters, explicitly revealing the impact of uncertainty in optimal decision-making [5]. This mathematical framework has led to breakthroughs in solving challenging formulations with uncertainty.

Differential geometry concepts have emerged as powerful tools for effective structural optimization and reliability analysis, providing strong mathematical representations and methods for examining intricate surfaces and forms [6]. The application of fundamental differential geometry concepts such as tensors, manifolds, and curvature, combined with probability theory in tangent spaces to account for geometrical uncertainties, enables the determination of structural reliability through uncertainty propagation [6]. By linking differential geometry representations with optimization techniques, researchers have enabled reliability-based design optimization [6].

For specific problems in nonsmooth dynamics involving set-valued forces, numerical methods such as the Alternating Direction Method of Multipliers (ADMM) have been developed to solve variational inequalities and cone complementarity problems. For frictional contacts, these methods can solve second-order cone complementarity problems, providing efficient and robust optimization techniques that draw on few computational primitives [7]. Researchers have reformulated original ADMM schemes to exploit the sparsity of constraint Jacobians and implemented optimizations such as warm starting and adaptive step scaling to improve computational performance [7].

The advantages of such mathematical approaches extend beyond uncertainty handling. Researchers have utilized multiparametric programming techniques to solve deterministic classes of problems by treating specific elements of the optimization program as uncertain parameters [5]. Recent years have witnessed a significant number of publications involving multiparametric programming, covering theoretical, algorithmic, and application developments, with potential for future contributions highlighting its benefits in research efforts [5].

In addressing constrained optimization problems, hybrid gradient simulated annealing algorithms have been guided by penalty function methods, which remain the most popular approach due to their simplicity and ease of implementation [8]. The simulated-annealing algorithm (SA) has emerged as one of the most successful meta-heuristic strategies, while gradient methods represent the most computationally inexpensive among deterministic approaches [8]. New approach penalty functions have been proposed to handle constraints and guide hybrid gradient simulated annealing algorithms in solving constrained optimisation problems [8].

## 3. Nature-Inspired Optimisation Algorithms

Nature-inspired optimization algorithms represent a significant category of techniques drawing inspiration from natural phenomena, biological behaviours, and physical processes. These algorithms have gained prominence due to their ability to handle complex, non-linear problems without requiring gradient information.

### 3.1 Bio-inspired Optimisation Algorithms

Bio-inspired algorithms form a substantial subset of nature-inspired optimization techniques, drawing on the behaviours and characteristics of living organisms.

The Hippopotamus Optimization (HO) algorithm represents a novel stochastic technique conceived by drawing inspiration from the inherent behaviours observed in hippopotamuses, showcasing an innovative approach in metaheuristic methodology [9]. The HO is conceptually defined using a trinary-phase model incorporating position updating in rivers or ponds, defensive strategies against predators, and evasion methods, all of which are mathematically formulated [9]. The algorithm demonstrated remarkable proficiency in both exploitation and exploration, attaining the top rank in 115 out of 161 benchmark functions in finding optimal values across various function types, while effectively balancing exploration and exploitation to support the search process [9].

Similarly, the Wild Horse Optimizer (WHO) simulates the social behaviour of wild horses in nature, though initial versions suffered from low exploitation capability and stagnation in local optima [10]. Improvements led to the Improved Wild Horse Optimizer (IWHO), which incorporated innovative strategies such as the random running strategy (RRS) to balance exploration and exploitation, and the competition for waterhole mechanism (CWHM) to boost exploitation behaviour [10]. The dynamic inertia weight strategy (DIWS) was further utilized to optimize the global solution [10].

The Monkey King Evolution (MKE) algorithm represents a population-based differential evolutionary algorithm where single evolution strategy and control parameters affect convergence and the balance between exploration and exploitation. Since evolution strategies significantly impact algorithm performance, collaborating multiple strategies can substantially enhance algorithmic capabilities [11]. The Multi-trial vector-based Monkey King Evolution (MMKE) algorithm introduces novel components such as best-history trial vector producer (BTVP) and random trial vector producer (RTVP) that effectively collaborate with canonical MKE through a multi-trial vector approach to tackle various real-world optimization problems with diverse challenges [11].

**Other significant bio-inspired algorithms include:**

The Remora Optimization Algorithm (ROA), which mimics the intelligent traveller behaviour of Remoras, with enhanced versions (EROA) addressing limitations like local optima stagnation and slow convergence [12].

The Grasshopper Optimization Algorithm (GOA), which mimics the biological behaviour of grasshopper swarms seeking food sources in nature, though the original version had shortcomings in global search ability and precision that required improvement [13].

The Portia Spider Algorithm (PSA), a swarm-based technique inspired by the unique predatory strategies of the Portia spider, which demonstrates the ability to tackle new challenges by learning and adapting strategies based on prior experiences [14].

The Mayfly Optimization Algorithm (MOA), a biomimetic metaheuristic algorithm with superior framework and optimization methods, though it has shortcomings in convergence speed and local optimization that have been addressed in enhanced versions such as AOBLMOA, which combines MOA with the Aquila Optimizer (AO) and opposition-based learning (OBL) strategy [15].

### 3.2 Physics-inspired and Human-inspired Algorithms

Beyond biological systems, optimization algorithms also draw inspiration from physical phenomena and human behaviours.

The Arithmetic Optimization Algorithm (AOA) is a metaheuristic approach that has been improved through population control strategies. By classifying the population and adaptively controlling the number of individuals in the subpopulation, the information from each individual can be used effectively to accelerate finding optimal values, avoid local optima, and improve solution accuracy [16].

The Election Optimizer Algorithm (EOA) draws inspiration from the democratic electoral system, focusing on the presidential election process to optimize solutions. By simulating the complete election process, EOA introduces a novel position-tracking strategy that expands the scope of effectively solvable problems, surpassing conventional human-based algorithms, specifically the political optimizer [17]. EOA incorporates explicit behaviours observed during elections, including party nomination and presidential election phases, where the search space is broadened to avoid local optima through diverse strategies, and adequate population diversity is maintained in later stages through further campaigning between elite candidates [17].

The Sine Cosine Algorithm (SCA) is widely recognized for its efficacy in solving optimization problems, though it encounters challenges in balancing exploration and exploitation. Advanced models such as the novel Sine Cosine Algorithm (nSCA) integrate mechanisms like roulette wheel selection (RWS) and opposition-based learning (OBL) techniques to enhance global optimization capabilities [18].

Human-Inspired Optimization Algorithms (HIOAs) represent another significant category, capitalizing on human intelligence and problem-solving abilities. Humans acquire high levels of intelligence with abilities to understand, reason, recognize, learn, innovate, retain information, make decisions, communicate, and solve problems. Human behavior and evolution empower humans to progress or acclimatize with their environments at rates exceeding other nature-based evolution approaches, creating an additional detachment from Nature-Inspired Optimization Algorithms [19].

## 4. Multi-objective Optimization Approaches

Multi-objective optimization (MOO) represents a crucial area in engineering optimization, addressing problems where multiple, often conflicting objectives must be simultaneously optimized.

The integration of multi-objective approaches with existing algorithms has expanded their applicability, as exemplified by the Multi-objective Exponential Distribution Optimizer (MOEDO), which extends the Exponential Distribution Optimizer (EDO) by introducing its multi-objective version, enhanced with elite non-dominated sorting and crowding distance mechanisms [20]. An information feedback mechanism (IFM) is integrated into MOEDO to balance exploration and exploitation, improving convergence and mitigating stagnation in local optima, a notable limitation in traditional approaches [20].

In the context of complex engineering systems, Multidisciplinary Design Optimization (MDO) has emerged as an advanced methodology to deal with problems involving multiple disciplines or subsystems with couplings or data interactions, with uncertainty being a crucial factor affecting system performance [21]. Novel Uncertain MDO (UMDO) methods based on conditional value at risk (CVaR) have been proposed, where an approximate method for CVaR under uncertainty risk analysis is derived and a UMDO framework based on CVaR is constructed [21]. These approaches reduce multidisciplinary analyses of complex systems using collaboration models and construct metamodels to simulate data interaction between multidisciplinary systems [21].

Large-scale Mult objective optimization problems (LSMOPs) represent a particular challenge, characterized by simultaneously optimizing multiple conflicting objectives and involving hundreds of decision variables. Many

real-world engineering applications can be modelled as LSMOPs, with engineering applications requiring insensitivity in performance, meaning algorithms should produce good results for every run with minimal fluctuation across multiple runs [22]. The large-scale Mult objective optimization algorithm via Monte Carlo tree search addresses these challenges by sampling decision variables to construct new nodes on the Monte Carlo tree for optimization and evaluation, selecting nodes with good evaluations for further searches to reduce performance sensitivity caused by large-scale decision variables [22].
Sustainability principles in structural design and decision-making processes for infrastructure have been integrated through a novel approach combining multi-objective optimization (MOO) with multi-criteria decision-making (MCDM) techniques, tailored specifically for the design and selection of reinforced concrete precast modular frames (RCPMF) [23]. The effectiveness of repair operators in optimizing economic, environmental, and social objectives is evaluated using a customized Non-dominated Sorting Genetic Algorithm II (NSGA-II), complemented by detailed life cycle analysis (LCA) [23].
For computationally expensive models in real-world engineering applications, surrogate-based MOO approaches have been developed. Despite significant computational power advances, conventional optimization procedures repeatedly invoked during the optimization process are hindered by computationally expensive models, leading to the use of surrogate models requiring far less time and resources [24]. The automated multiobjective surrogate-based Pareto finder MOO algorithm (AMSP) reduces the cost of fitness function evaluations and locates the Pareto frontier by creating three surrogate models using data samples from the feasible design region, then iteratively sampling and updating the Pareto set by assigning weighting factors to surrogates based on root mean squared error values [24].

## 5. Constraint Handling Techniques

Constraint handling represents a critical aspect of optimization in engineering, as most real-world problems involve various constraints that limit the feasible solution space.
Scholarly literature on constraint-handling techniques for single-objective and multi-objective population-based algorithms has been analyzed according to the most relevant journals and articles, reviewing the main ideas of state-of-the-art constraint handling techniques in population-based optimization, followed by bibliometric analysis focused on multi-objective approaches [3]. Analysis of research published between 2000 and 2021 indicates that constraint-handling techniques for multi-objective optimization have received much less attention compared to single-objective optimization [3]. The most promising algorithms for such optimization were determined to be genetic algorithms, differential evolutionary algorithms, and particle swarm intelligence, with engineering, computer science, and mathematics identified as the top three research fields anticipated to see increased future research [3].
In algorithms like the Butterfly Optimization Algorithm (BOA), which considers only smell perception rules, there is a tendency to fall into local optima. Enhanced versions such as the Hybrid-Flash Butterfly Optimization Algorithm (HFBOA) incorporate additional operators like color perception rules to better align with actual foraging characteristics of butterflies in nature [25]. Updating strategies for control parameters using logistic mapping further enhances global optimal ability [25]. When applied to engineering constrained optimization problems such as tubular column design, tension/compression spring design, and cantilever beam design, these enhanced approaches demonstrate superior performance in solving complex real-world engineering constrained tasks [25].
In addressing constrained optimization problems using deterministic, stochastic optimization methods, or hybridization between them, penalty function methods remain the most popular approach due to their simplicity and ease of implementation, with many approaches available for handling constraints [8]. The hybrid gradient simulated annealing algorithm (GLMSA), which has demonstrated efficiency and effectiveness in solving unconstrained optimization problems, has been generalized to address constrained optimization problems through new approach penalty functions proposed to handle constraints and guide the algorithm, resulting in the GHMSA algorithm [8].
For specific applications like project risk management, comprehensive methodologies incorporate constraint handling through various techniques. After collecting and categorizing risks using brainstorming, validating them with Fuzzy Delphi techniques, determining relationships through Interpretive Structural Modelling (ISM), and calculating criteria weights with Fuzzy Best-Worst Method, critical risks can be identified using the fuzzy WASPAS method [26]. A bi-objective mathematical programming model can then be developed and solved using the Augmented Epsilon-Constraint (AEC) method to determine optimal risk response strategies for each critical risk, demonstrating effectiveness in managing construction project risks [26].

## 6. Hybrid and Enhanced Optimization Methods

Hybrid optimization methods, which combine multiple algorithms or enhancement strategies, have emerged as powerful approaches to overcome limitations of individual algorithms and improve overall performance.

To address limitations in algorithms like the Remora Optimization Algorithm (ROA), which may get stuck in local optimal regions or have slow convergence in high-dimensional complicated problems, improved versions such as Enhanced ROA (EROA) have been developed using adaptive dynamic probability, SFO with Levy flight, and restart strategy [12]. The performance of these enhanced algorithms tested on different benchmarks and real-world engineering problems demonstrates their efficiency through statistical analysis and experimental results [12].

The Multi-trial vector-based Monkey King Evolution algorithm (MMKE) was designed to improve global search capability, balance exploration and exploitation, and prevent premature convergence during the optimization process [11]. When assessed using CEC 2018 test functions and compared with eight metaheuristic algorithms, MMKE demonstrated competitive and superior results in terms of accuracy and convergence rate, with Friedman test statistical analysis confirming its significant superiority [11].

In the domain of energy performance analysis, hybrid machine learning models like feed-forward neural networks (FFNN) optimized via the electrostatic discharge algorithm (ESDA) have been developed for predicting parameters such as annual thermal energy demand and annual weighted average discomfort degree-hours in residential buildings [27]. These hybrid approaches demonstrate superior accuracy compared to similar optimization techniques such as atom search optimization (ASO), future search algorithm (FSA), and satin bowerbird optimization (SBO), with ESDA-FFNN achieving higher Pearson correlation indices [27].

The Sine Cosine Algorithm (SCA), while widely recognized for its efficacy in solving optimization problems, faces challenges in balancing exploration and exploitation. The novel Sine Cosine Algorithm (nSCA) addresses these limitations by integrating roulette wheel selection (RWS) mechanism and opposition-based learning (OBL) techniques to augment its global optimization capabilities [18]. When compared with state-of-the-art optimization algorithms across classical test functions, CEC2017 benchmark functions, and engineering optimization case studies, nSCA consistently outperforms competitors, providing more effective solutions to both theoretical and applied optimization problems [18].

The Improved Arithmetic Optimization Algorithm (IAOA) based on population control strategy addresses numerical optimization problems by classifying the population and adaptively controlling the number of individuals in the subpopulation. This approach effectively utilizes information from each individual, accelerating optimal value discovery, avoiding local optima, and improving solution accuracy [16]. When evaluated on systems of nonlinear equations, integrations, and engineering problems, IAOA outperforms other algorithms in convergence speed, accuracy, stability, and robustness [16].

The Modified Sine Cosine Algorithm (MSCA) redefines the position update formula of SCA to increase convergence speed and adopts the Levy random walk mutation strategy to improve population diversity [28]. Verification against classical benchmark problems, IEEE CEC2017 test suites, and complex engineering design problems demonstrates MSCA's good convergence, robustness, and engineering utility [28].

According to the Non-Free Lunch theorem, no single algorithm can solve all optimization problems, necessitating hybrid approaches. The novel hybrid algorithm EAOAHHO combines the Arithmetic Optimization Algorithm (AOA) and Harris Hawks Optimization (HHO) framework to more efficiently solve industrial engineering design problems [29]. Introduction of pinhole imaging opposition-based learning increases population diversity and capability to escape local optima, while composite mutation strategy enhances exploitation and exploration for better convergence accuracy [29].

The improved grasshopper optimization algorithm (CMRWGOA) combines both Random Weight (RWGOA) and Cauchy mutation (CMGOA) mechanisms with the GOA inspired by grasshopper foraging and swarming habits. Its performance has been validated against benchmark functions, with non-parametric statistical tests confirming its effectiveness, particularly when applied to real-life challenging optimization problems [30].

## 7. Applications in Engineering Design

Optimization techniques have found extensive applications across various engineering domains, enabling more efficient, sustainable, and effective designs and processes.

The Hippopotamus Optimization (HO) algorithm has effectively addressed four distinct engineering design challenges, achieving the most efficient resolution while concurrently upholding adherence to designated constraints [9]. When compared with widely researched metaheuristics like WOA, GWO, SSA, PSO, SCA, FA, GOA, TLBO, MFO, and IWO, as well as recently developed algorithms like AOA and high-performance optimizers like CMA-ES, statistical post hoc analysis determined HO to be significantly superior [9].

The Improved Wild Horse Optimizer (IWHO) has been evaluated using classical benchmark functions, CEC 2021 test functions, and real-world optimization problems, including high-dimensional cases (D = 200, 500, 1000). Compared with nine well-known algorithms, experimental results demonstrate IWHO's competitiveness in convergence speed, precision, accuracy, and stability, with practical capability verified through engineering design problems [10].

In construction project management, the analysis of time-cost relationships is crucial, with various optimization techniques developed to solve time-cost trade-off problems. The hybrid multi-verse optimizer model (hDMVO), which combines the multi-verse optimizer (MVO) and sine cosine algorithm (SCA), addresses the discrete time-cost trade-off problem (DTCTP) [31]. When evaluated against 23 benchmark test functions, hDMVO demonstrates competitiveness with MVO, SCA, the dragonfly algorithm, and ant lion optimization [31]. When applied to benchmark DTCTP test problems, including medium-scale (63 activities) and large-scale (630 activities) instances, hDMVO provides superior solutions for time-cost optimization in large-scale and complex projects compared to previous algorithms [31].

Optimization modelling of technological processes for surface treatment, particularly anodizing, has received limited attention despite the multitude of articles devoted to mathematical optimization methods for controlling technological processes. This gap has stimulated research to apply non-linear programming (NLP) methods to optimize anodic oxidation of aluminum using MATLAB toolboxes [1]. The novelty in this application lies in selecting effective approaches for optimal process conditions, proposing a solving strategy based on design of experiments, exploratory data analysis, confirmatory analysis, and optimization modelling, with the main contribution being a mathematical-statistical computational model predicting the thickness of the aluminum anodic oxide layer to achieve thicknesses required by clients [1].

Structural optimization has been employed to counter the high environmental impact associated with concrete as the most widely used construction material, bridging the gap between structural engineering and mathematical optimization through design competitions. Design concepts for optimized concrete girders based on strut-and-tie modelling have been investigated within competition boundaries [32]. Results indicate the importance of adequate structural interpretation of topology optimization results for satisfying structural performance, with reinforcement mass having a substantial share in total Global Warming Potential, and successful numerical re-simulation serving as a modelling base for other researchers, achieving over 30% increase in resource efficiency compared to conventionally designed girders [32].

In manufacturing, electric discharge machining (EDM) is essential for machining porous sintered metals like 316L porous stainless steel (PSS), which has excellent lightweight and damping properties along with superior mechanical and metallurgical properties. Conventional machining techniques are unsuitable as they tend to block micro-pores, reducing breathability, making EDM an effective alternative [33]. Meta-heuristic optimization techniques such as Teaching Learning-Based Optimization (TLBO) and Particle Swarm Optimization (PSO) algorithms have been applied to optimize machining process parameters to maximize material removal rate and minimize tool wear rate, with PSO demonstrating consistent improvements over TLBO and faster convergence with minimal computational resources [33].

In biomedical engineering, intelligent design of nerve guidance conduits (NGCs) has employed artificial intelligence-driven fluid structure interaction studies for modelling and optimizing nerve growth. These conduits are effective in promoting nerve regeneration in clinical applications including trauma or surgery-induced nerve defects, though challenges exist in repair length, natural nerve replication, and substance degradation affecting neurotrophic factor delivery [34]. Mathematical surrogate models from data-based modelling have been used with AI optimization algorithms like differential evolution (DE) and non-dominated sorting genetic algorithm II (NSGA-II), revealing that both algorithms generate nearly identical solutions, with significant improvements in physical properties like porosity and stress distribution through optimal design parameters [34].

## 8. Performance Evaluation and Benchmark Studies

Performance evaluation through benchmark studies plays a crucial role in assessing and comparing different optimization algorithms, providing insights into their strengths, weaknesses, and suitability for various problems. The Hippopotamus Optimization (HO) algorithm's performance was extensively evaluated across 161 benchmark functions, including unimodal and high-dimensional multimodal functions, fixed-dimensional multimodal functions, CEC 2019 test suite, CEC 2014 test suite with dimensions of 10, 30, 50, and 100, and Zigzag Pattern benchmark functions, attaining the top rank in 115 out of 161 functions in finding optimal values [9].

The Saltatory Evolution Ant Colony Optimization (SEACO) algorithm demonstrated superior performance compared to traditional ACO algorithms through extensive experiments on traveling salesman problem library (TSPLIB) database, with better solution quality and greater suitability for large-scale datasets within specified time windows [35].

The enhanced Grasshopper Optimization Algorithm with Levy Flight (LFGOA) was tested using 23 mathematical benchmark functions against eight well-known meta-heuristic algorithms and seven real-world engineering problems, with statistical analysis and experimental results confirming its efficiency and potential as an alternative for meta-heuristic optimization problems due to its high exploration and exploitation capabilities [13].

The novel Sine Cosine Algorithm (nSCA) underwent meticulous evaluation against state-of-the-art optimization algorithms, including multi-verse optimizer (MVO), salp swarm algorithm (SSA), moth-flame optimization (MFO), grasshopper optimization algorithm (GOA), whale optimization algorithm (WOA), and the original SCA, across 23 classical test functions and 29 CEC2017 benchmark functions, as well as five engineering optimization case studies, consistently outperforming competitors in both theoretical and applied optimization problems [18].
The Quantum Chimp Optimization Algorithm (QU-ChOA) was evaluated across diverse domains, including benchmark tests, the IEEE CEC-06–2019 100-Digit Challenge, real-world optimization problems from IEEE-CEC-2020, and dynamic scenarios from IEEE-CEC-2022, achieving an average success rate of 88.98% across various benchmark functions and demonstrating robust global search abilities with 14,012 average fitness evaluations and an average calculation duration of 58.22 units in fire detection applications [36]. In benchmark tests, QU-ChOA outperformed traditional algorithms, achieving a perfect success rate of 100% in several functions of the IEEE CEC-06–2019 100-Digit Challenge, while real-world applications highlighted significant improvements in objective function values for industrial processes [36].
The Election Optimizer Algorithm (EOA) underwent rigorous assessment against renowned optimization algorithms across twenty-three standard test functions and CEC2019, demonstrating superior performance in terms of average values and standard deviations, with Wilcoxon rank-sum statistical analysis at a significance level of 0.05 confirming EOA's consistent delivery of high-quality solutions compared to benchmark algorithms [17].
The multi-strategy integrated sand cat swarm optimization algorithm (MSCSO) was evaluated using 29 benchmark functions of 30, 50, and 100 dimensions from CEC 2017, with Wilcoxon rank-sum tests and Friedman's test verifying its global solid search ability and convergence performance [37]. In practical engineering problems like reducer and welded beam design, MSCSO demonstrated superior performance compared to five other intelligent algorithms, showing remarkable ability to approach optimal solutions [37].
The improved Coati Optimization Algorithm based on chaotic sequence, nonlinear inertia weight, adaptive T-distribution variation strategy, and alert updating strategy (TNTWCOA) was evaluated using IEEE CEC2017 with 29 classic test functions and applied to engineering design optimization problems such as pressure vessel optimization and welding beam design, with results compared against various algorithms including ICOA, COA, GJO, OOA, SCSO, and SABO, demonstrating significantly improved convergence speed, optimization accuracy, and robustness [38].
The hybrid biomimetic optimization algorithm AOBLMOA, combining the Mayfly Optimization Algorithm, Aquila Optimizer, and opposition-based learning strategy, was verified through three experiment sets: testing optimal strategy selection among multiple combined strategies on 19 benchmark functions, evaluation against state-of-the-art metaheuristic algorithms using 30 CEC2017 numerical optimization problems, and application to 10 CEC2020 real-world constrained optimization problems, demonstrating effectiveness, superiority, and feasibility in both numerical optimization and engineering design problems [15].

## 9. Current Challenges and Future Directions

Despite significant advances in optimization techniques for engineering applications, several challenges and research gaps persist, pointing to future research directions and emerging trends.
Bibliometric analysis of constraint-handling techniques indicates that multi-objective optimization has received substantially less attention compared to single-objective optimization, suggesting a significant gap in research that needs to be addressed [3]. The rapid proliferation of optimization algorithms (over 300 new methodologies in the last decade) highlights the need for sophisticated understanding of these novel methods, with extensive comparative analyses against conventional benchmarks like search history, trajectory plots, and fitness functions necessary to elucidate the superiority of new approaches and provide directions for refining and expanding upon these methodologies [2].
In the context of derivative-free optimization (DFO) for complex systems, algorithm selection remains a nonintuitive task due to uncertain performance, particularly when system evaluation cost or computational load is high (e.g., density functional theory and computational fluid dynamics), making the quick convergence to near-global optima in early optimization stages critically important [39]. Comparative analysis of various optimization algorithms—including deterministic global search, global model-based search, metaheuristic, and Bayesian optimization—applied to benchmark problems and real-world scenarios like energy process optimization reveals significant performance variations based on problem types and variable numbers [39].
In large-scale multiobjective optimization, engineering applications require performance insensitivity, meaning algorithms should produce good results consistently with minimal fluctuation across multiple runs, yet existing algorithms often focus on improving performance while paying little attention to insensitivity characteristics, leading to substantial limitations in solving practical problems [22]. The development of metrics to measure algorithm sensitivity and comparative analysis on different benchmark functions and metrics represents a promising research direction [22].

As engineering optimization problems increase in complexity, classical gradient-based optimization algorithms face limitations in their problem-solving capacity, while metaheuristics have gained popularity due to their simplicity and result robustness, with population-based bio-inspired algorithms demonstrating favorable performance across various optimization problems [40]. Comprehensive reviews of specific algorithms, their variations, and applications provide valuable insights for developing modified versions and hybridizations to improve upon original algorithms and existing variants, helping researchers develop superior metaheuristic optimization algorithms with recommendations for add-on intelligent agents [40].
The rapid expansion of algorithm families, particularly Human-Inspired Optimization Algorithms (HIOAs), makes it challenging for researchers to select appropriate algorithms and map problems accordingly, requiring proper understanding of theoretical fundamentals, governing rules, and common structures, with common challenges and open research issues needing careful consideration [19]. Efforts to categorize algorithms based on various criteria and analyze their building blocks can help achieve these objectives, providing a more organized framework for algorithm selection and application [19].

## 10. Conclusion

This comprehensive literature review has explored optimization techniques in engineering from a mathematical perspective, covering mathematical foundations, algorithm classifications, multi-objective approaches, constraint handling methods, hybrid techniques, engineering applications, performance evaluation methodologies, and future research directions.
The mathematical foundations of optimization techniques provide the theoretical underpinnings for their effective implementation, with approaches like multiparametric programming, differential geometry concepts, and mathematical reformulations enabling more efficient problem-solving in complex engineering contexts. These mathematical foundations have facilitated the development of diverse optimization algorithms, from nature-inspired techniques mimicking biological behaviours and physical processes to sophisticated hybrid approaches combining multiple methodologies.
Nature-inspired optimization algorithms, including bio-inspired techniques like the Hippopotamus Optimization algorithm, Wild Horse Optimizer, and Monkey King Evolution algorithm, along with physics-inspired and human-inspired approaches such as the Arithmetic Optimization Algorithm, Election Optimizer Algorithm, and Sine Cosine Algorithm, have demonstrated remarkable capabilities in solving complex engineering problems. The evolution of these algorithms through enhancement strategies and hybridization has further improved their performance, addressing limitations like premature convergence and local optima entrapment.
Multi-objective optimization approaches have expanded the applicability of optimization techniques to problems with multiple competing objectives, with methods like MOEDO, UMDO based on CVaR, and NSGA-II demonstrating effectiveness in various engineering contexts. Constraint handling techniques have similarly evolved, with penalty function methods, hybrid approaches, and specialized mechanisms enabling the effective navigation of constrained solution spaces.
The application of these optimization techniques across diverse engineering domains—from structural design and manufacturing processes to energy systems and biomedical engineering—has yielded significant improvements in efficiency, sustainability, and performance. Rigorous performance evaluation through benchmark studies has provided valuable insights into algorithm strengths, weaknesses, and suitability for different problem classes.
Despite significant advances, challenges remain in areas such as the limited research on multi-objective constraint handling, algorithm selection for computationally expensive problems, and achieving performance insensitivity in large-scale optimization. Future research directions include the development of more sophisticated hybrid algorithms, improved performance metrics, and more structured frameworks for algorithm selection based on problem characteristics.
As optimization techniques continue to evolve, their integration with artificial intelligence, machine learning, and advanced computing paradigms promises to further enhance their capabilities and applications in engineering, driving innovation and improved performance across disciplines.

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