International Journal of Interdisciplinary Innovative Research & Development (IJIIRD)
ISSN: 2456-236X
Vol. 10 Issue 01 | 2025

The New Algorithm for solving $\frac{dy}{dt} = g(t)$,where g(t) is trigonometric power function with BVP

Prof.Pragati A. Choudhari[1], Prof.Madhukar T. Tekade[2], Prof.B.A.Narkhede[3], Dr. Rajkumar P.Narkhede[4], Dr.N.O.Warade[5]

[1,2,3] Assistant Professor, Applied Science and Humanities, Padm.Dr.V.B.Kolte college of engineering, Malkapur, Maharashtra, India

[4] Assistant Professor, Applied Science and Humanities Konkan Gyanpeeth college of engineering, Karjat, Maharashtra, India

[5] Assistant Professor, Applied Science and Humanities, MGM College Of Engineering,Kamothe, Panvel, Navi - Mumbai, Maharashtra, India

DOI: 10.5281/zenodo.15740706

ABSTRACT

The ZDTM-Zhou's differential transform method is approximation method which construct analytical series solution of linear and non-linear BVP which reduces the computational work than Taylor's series for higher order linear and nonlinear Homogenous or non Homogeneous differential equations with initial conditions. Or non homogenous differential equations with initial conditions. In this paper ZDTM is used to find numerical solutions of the linear ordinary differential equations of first order and first degree of type

Y1 (t) = g (t) with y (u) = $\propto$

Where t (t) is power function of trigonometric or Hyperbolic functions to find transformation of such functions we have use ZDTMNPC – Zhous differential transformation method by Narhari Theorem which also have wide applications for BVP f(D)Y = g (t)

Keyword: **-** ZDTM, ZDTMNPC ordinary differential equations Initial value problem.

1. INTRODUCTION

ZDTM-Zhou's differential transform method is nothing but easy generalization of Taylor's series method to solve linear and non linear Homogeneous as well as non homogeneous differential equations with IVP till to day there is no ZDTM results for transformation of trigonometric & Hyperbolic functions

Nomenclature

ZDTM- Zhous Differential Transform Method

ZDTMNPC- Zhous Differential Transform Method Narharis Principal Corollery

in terms of powers which we can arly transform by using ZDTMNPC to solve with BVP with some ZDTM Theorems.

G.E. Pukhov used Taylor series transformation for solving differential equation[1], later on he did work on expansion formula for differential transformation method[2], G.E. Pukhov also done work on differential transforms of functions and equations[3], Zhou J. K., ZDTM method and its applications for electrical circuits problems[4], Chiou et.al. applied Taylor series transform to nonlinear vibration problems[5], Chen C.J. et.al used ZDTM to obtained solution differential equation damped vibration of hard and soft spring[6], Jorba A. and M. Zou has been used high order Taylor series method for the numerical integration(7], I.H.A.H. Hassan have find solution of Higher order initial value problems of differential equation by ZDTM[8], Jang M.J. et.al solve initial value problems by using Zhous differential transform method[9], F. Ayaz find solutions of differential equations initial value problems by ZDTM[10], Kurnaz A. al. used ZDTM in n-dimension for solving partial differential equations I.V.P. [11], F. Kangalgi and Ayaz F. have applied ZDTM for finding solutions of linear and non-linear heat equations[12], A.S.V. Ravikanth and K. Aruna find out solutions of linear and non-linear Klein-Gordon equation[13], Khaled Batiha used ZDTM to obtain solution of linear and non linear ODE[14], A. Gokdogan et.al used ZDTM multistage for approximate solution of Hantavirus infection model[15], Rashidi M.M. et.al studied application of multi-step ZDTM on shrinking sheet[16], Z.M. Odibat, S. Momani and V.S. Erturk differential equation by generalized ZDTM[17], V.S. Erturk et.al used ZDTM to multi order fractional differential equations[18].

2. DEFINITION OF ZDTM

An arbitrary function g (t) can be expanded in Taylor series about t = 0

$$g(t) = \sum_{K=0}^{\infty} \frac{t^k}{K!}\left[\frac{d^k g}{dt^k}\right]_{t=0}$$

DTM of g (t) is

$$G(k) = \frac{1}{K!}\left[\frac{d^k g}{dt^k}\right]_{t=0}$$

3.1 THEOREMS ON DTM METHOD

Original Function		Transformation
1)	$g(t) = x(t) + y(t)$	$G(K) = X(k) + Y(k)$
2)	$g(t) = \propto x(t)$	$G(K) = \propto X(k)$
3)	$g(t) = \frac{d}{dt} x(t)$	$G(K) = (k+1)\, X(k+1)$
4)	$g(t) = \frac{d^m}{dt^m} x(t)$	$G(K) = (k+1)(k+2)\ldots(k+m)\, X(k+m)$
5)	$g(t) = t^m$	$G(K) = \delta(k-m)$ $= 1 \quad k = m$ $= 0 \quad k \neq m$
6)	$g(t) = e^{\lambda t}$	$G(K) = \frac{\lambda^k}{k!}$
7)	$g(t) = \mathrm{Sin}(\propto t + \beta)$	$G(K) = \frac{\propto^k}{k!} \mathrm{Sin}\left(\frac{k\pi}{2} + \beta\right)$
8)	$g(t) = \mathrm{Cos}(\propto t + \beta)$	$G(K) = \frac{\propto^k}{k!} \mathrm{Cos}\left(\frac{k\pi}{2} + \beta\right)$
9)	$g(t) = x(t)\,.\,y(t)$	$P(K) = \sum_{m=0}^{K} Y(m)\, X(k-m)$
10)	$g(t) = (1+t)^m$	$G(K) = \frac{m(m-1)\ldots\ldots(m-k+1)}{K!}$

3.2. Theorems on ZDTMNPC

1) It $n(t) = (\sin^4 t + \cos^4 t)$ then $N(K) = \frac{3}{4}\,\delta(K) + \frac{4^{K-1}}{K!}\,\mathrm{Cos}\left(\frac{K\pi}{2}\right)$

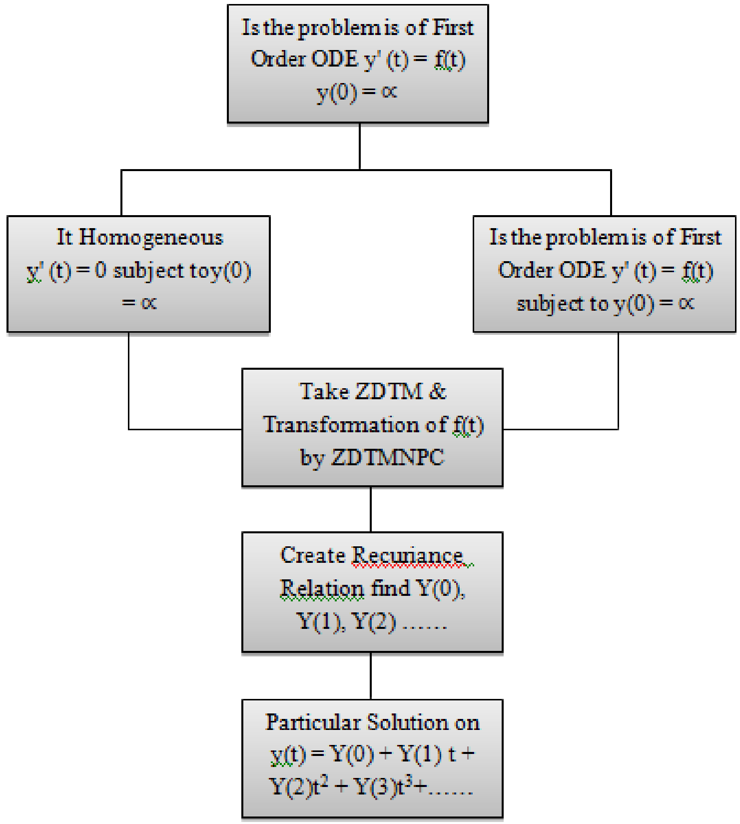
2) It $n(t) = (\cos t + i \sin t)^n$ then $N(K) = \frac{n^K}{K!} e^{iK\pi/2}$

3) It $n(t) = (t\cos^2 t + t\sin^2 t)^2$ then $N(K) = \delta(K-2)$ where

$= 0 \quad K \neq 2$

$= 1 \quad K = 2$

4. Flow chart of ZDTMNPC – for first order L.D.E. initial value problems



---

5. Experimentation of ZDTMNPC results

Example : 1

Solve by using ZDTMNPC

)

$y'(t) = (t\cos^2 t + t\sin^2 t)^2$

→ By ZDTMNPC

$(K + 1)\, Y(K + 1) = \delta\,(K - 2)$

Put $K = 0, 1, 2, 3, 4$ ………………….

$Y(0) = 0$

$Y(1) = 1$

$Y(2) = 0$

$Y(3) = 1/3$

$Y(4) = 0$

Solution is given by

$y(t) = Y(0) + t\,Y(1) + t^2 Y(2) + t^3 Y(3) +$ ……………….

$= \frac{t^3}{3}$

which is exact solution of above IVP. Exact solution is same as ZDTMNPC solution.

Example : 2

Solve by using ZDTMNPC

$y'(t) = (\sin^4 t + \cos^4 t)$

with $y(0) = 0$

→ By using ZDTMNPC - Results

$$(K+1)\,Y(K+1) = \frac{3}{4}\delta(K) + \frac{4^{K-1}}{K!}\cos\left(\frac{K\pi}{2}\right)$$

Put $K = 0, 1, 2, 3, 4$ …………………

$Y(0) = 0$

$Y(1) = 1$

$Y(2) = 0$

$Y(3) = 2/3$

$Y(4) = 0$

$Y(5) = 8/15$

$Y(6) = 0$

$Y(7) = \frac{-64}{315}$

Solution is given by

$y(t) = Y(0) + Y(1)t + Y(2)t^2 + Y(3)t^3 +$ ………………

$= t - \frac{2}{3}t^3 + \frac{8}{15}t^5 - \frac{64}{315}t^7 +$ ………………………

Whose exact solutions

$$y(t) = \frac{3}{4}t + \frac{1}{16}\sin 4t$$

$$y(t) = \frac{3}{4}t + \frac{1}{16}\left\{(4+) - \frac{(4+)^3}{3!} + \frac{(4+)^5}{5!} - \cdots\right\}$$

$$= \frac{3}{4}t + \frac{t}{4} - \frac{2}{3}t^3 + \frac{8}{15}t^5 - \frac{64}{315}t^7 + \ldots\ldots$$

$$= t - \frac{2}{3}t^3 + \frac{8}{15}t^5 - \frac{64}{315}t^7 + \ldots\ldots$$

Exact solution is same as by using ZDTMNPC results

Example : 3

Solve $z'(t) = (\cos t + i \sin t)^n$

with $Z(0) = 0 - \frac{1}{n}i$

Where

→ $z(t) = x(t) + iy(t)$

$z'(t) = x'(t) + iy'(t)$

$x(0) = 0$

$y(0) = -\frac{1}{n}$ etc

By ZDTMNPC - Results

(K + 1) Z(K + 1) = $\frac{n^K}{K!} e^{i\left(\frac{K\pi}{2}\right)}$

Put K = 0, 1, 2, 3, 4 …………………

Z (0) = $-\frac{1}{n}\, i$

Z (1) = 1

Z (2) = $\frac{n}{2!}\, i$

Z (3) = $\frac{-n^2}{3!}$

Z (4) = $\frac{n^3}{4!}\,(-i)$

Z (5) = $\frac{n^4}{5!}$

Series solution is given by

z(t) = Z(0) + Z(1)t + Z(2) t^2 + Z(3) t^3 + ………………

= -i + nt + $\frac{n^2t^2}{2!} i - \frac{n^3t^3}{3!} - \frac{n^4t^4}{4!} i - \frac{n^5t^5}{5!}$ + ………..

= $\frac{1}{n}$[Sin nt – i cos nt]

Which exactly same as exact solution.

6. Validation and Comparison

The ZDTM has promising approach by using ZDTMNPC results in various field of science and engineering which gives best approximation reliable than other existing method. ZDTMNPC - Results have very wide scope for solving initial value problems of higher order LDE.

7. CONCLUSION

In this work ZDTM and ZDTMNPC applied for initial value problems of ordinary differential equation for first order and first degree in which exact solution is compared with ZDTM solution which does not require any kind of strong assumptions by considering maximum terms in series solution we get more accuracy by ZDTM.

At the same time ZDTMNPC are powerful tools for solving B.V.P. of type $\frac{d^n y}{dt^n} = f(t)$ where f(t) is in terms of powers of trigonometric or Hyperbolic functions. Similar results and also valid for complex variable in case of total differential.

At the same time ZDTMNPC are powerful tools for solving B.V.P. of type $\frac{d^n y}{dt^n} = f(t)$ where f(t) is in terms of powers of trigonometric or Hyperbolic functions. Similar results and also valid for complex variable in case of total differential.

Acknowledgements

I would like to thank our colleagues from Datta Meghe College of Engineering who provided insight and expertise that greatly assisted the research.

REFERENCES

1. G. E. Pukhov, Computational structure for solving differential equations by Taylor transformations, Cybern. Syst. Anal. 14 (1978) 383.
2. G. E. Pukhov, Expansion formulas for differential transforms, Cybern. Syst. Anal. 17 (1981) 460.
3. G. E. Pukhov, Differential transforms of functions and equations, in Russian, Naukova Dumka, Kiev, 1980.
4. J. K. Zhou, Differential transformation and its applications for electrical circuits, in Chinese, Huarjung University Press, Wuuhahn, China, 1986.
5. J. S. Chiou and J. R. Tzeng, Application of the Taylor transform to nonlinear vibration problems, J. Vib. Acoust. 118 (1996) 83.
6. C.-.J. Chen and W.-J. Wu, Application of the Taylor differential transformation method to viscous damped vibration of hard and soft spring system, Comput, Struct. 59 (1996) 631.
7. A. Jorba and M. Zou, A software package for the numerical integration of ODEs by means of high-order Taylor methods, Exp. Math. 14 (2005) 99.
8. H. A.-H. Hassan, Differential transformation technique for solving higher order initial value problems , Appl. Math. Comput. 154 (2004) 299.
9. M. J. Jang, C.-L. Chen and Y.-C. Liu, Two-dimensional differential transform for partial differential equations, Appl. Math. Comput. 121 (2001) 261.
10. M. M. Rashidi, A. J. Chamkha and M Keimanesh, Application of multistep differential transform method on flow of a second grade fluid over a stretching or shrinking sheet, Am. J. Comput. Math. 1 (2011) 119.
11. A. Gokdogan, M. Merdan and A. Yildirim, A multistage differential transformation method for approximate solution of Hantavirus infection model, Commun. Nonlinear Sci. 17 (2012) 1.
12. V. S. Erturk, Z. M. Odibat and S. Momani, An approximate solution of a fractional order differential equation model of human T-cell lymphotropic virus I (HTLV-I) infection of T-cells, Comput. Math, Appl. 62 (2011) 996.
13. F. Ayaz, Solutions of the system of differential equations by differential transform method, Appl. Math. Comput. 147 (2004) 547.
14. A. Kurnaz, G. Oturang and M. E. Kiris, n-Dimensional differential transformation method for solving PDEs, Int. J. Comput. Math. 82 (2005) 369.
15. F. Kangalgil and F. Ayaz, Solution of linear and nonlinear heat equations by differential Transform Method , Selguk J. Appl. Math8 (2007) 75.
16. A.S.V. Ravi Kanth and K. Aruna, Differential transform method for solving the linear and nonlinear Klein-Gordon equation, Comput. Phys. Commun. 180 (2009) 708.
17. S. Momani, Z. M. Odibat and V.S. Erturk, Generalized differential transform method for solving a space and time-fractional diffusion-wave equation, Phys.Lett. A 370 (2007) 379.
18. A. Arikoglu and I. Ozkol, Solution of fractional integro-differential equations by using fractional differential transform method, Chaos, Solitons and Fractals 40 (2009) 521.