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How Engineers deal with Mathematics While Solving Differential Equation

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ABSTRACT

Numerical methods are tools for approximating solutions to problems that may have complicated developments or cannot be solved analytically. In engineering courses, students must address problems related other disciplines such as structural or rock mechanics, chemistry or physics. Prior to solving these problems it is important to define and adopt a rational framework. The students of fourth course out of five, of the bachelor's degree in Computer Engineering and Electrical Engineering at the University of Mumbai, they learn mathematics solving real problems with the help of the already acquired interdisciplinary knowledge. We have proposed the students a term project that summarizes some of the knowledge and competences acquired during the lessons. We will describe in this study the software and specific applications that will be use during the whole course.

Keywords: Numerical methods, differential equations, analytical solutions

1 Introduction

Contrary to students' belief, mathematics is rich in applications to the modern and real world. There is a need to convince students that mathematics is a relevant and living subject, with exciting applications in different areas of activity which affect their daily lives. Using mathematical tools, engineering students can look forward to a wide range of working fields in emerging technologies and modern application areas.

The ability to use the computer is an important part of the Numerical Methods course. In order to make students develop their own self-learning capabilities, they must produce a written and an oral presentation which analyzes an issue that requires the use of knowledge and competences acquired during the course. Scilab/Matlab program is one of the preferred software for engineers. It is a language for technical computing that includes an easy-to-use environment where problems and solutions are exist

The Numerical Methods course presents numerical methods, including their implementation into computers, and their application to the solution of some problems such as equilibrium of solids, heat transfer or water seepage through soils. The main topics goes from an initial overview of different numerical methods for one variable functions, to partial differential equations and finite elements method.

To assess students learning process, they have different homework assigned weekly that must be done in a defined period of time. The homework lies in solving some problems and questionnaires using Scilab /Matlab or making them handwritten. As part of the learning process, the virtual learning environment that we use as a corporative tool, supposes a great advantage, providing powerful capabilities for learning.

Engineering students use to separate the matters in self-contained compartments, so they could not see the relation between mathematics, electricity installations, process engineering, or material structures for example. There are several publications about specific tools for engineering, see for example the virtual campus for Biomedical Engineering a software application used for the study of heat transfer problems, a computer based educational tool for DC–DC converters and so on. This paper tries to put together both: the computer tools and the mathematical competences and abilities to obtain the solutions of some numerical methods' problems. We present a case study about a second-order ordinary differential equation that models a population evolution.

This paper is organized as follows: the Numerical Methods' course outline is detailed in Section 2. Section 3 is dedicated to the project proposal where students should choose the right tool. The assessment process is shown in Section 4, and finally, Section 5 draws the conclusions.

2 Numerical Methods: Course Outline

The Numerical Methods subject at the Computer Sciences Faculty and Industrial Engineering School at the University of Mumbai is a course in differential equations. Its goal is to find the solutions of equations involving derivatives of functions. Students will study:

x First order differential equations;
x Second-order differential equations;
x Applications in engineering and models;

There are no defined pre-requisites for this course, but students should have good grounding in differentiation and theory of integration, and they should have acquired sufficient basic integration skills. Some of the objectives of this course are:

1. Write and clearly express the mathematical notation;
2. Use computer technology to solve differential equations and interpret their results;
3. Acquire the needed knowledge about differential equations as a problem-solving tool;
4. Find solutions of quadratic linear differential equations and also first order;
5. Use different methods: numerical, graphical, and symbolic to solve problems;
6. Find answers to real-world problems applying the new mathematical knowledge of differential equations.

After successfully completing this course the students should achieve the following competences:

1. First-order differential equations:
a. Find the solution to linear first-order equations.
b. Solve initial value problems.
c. Determine whether a given first-order initial value problem has a unique solution.
2. Second-order differential equations:
a. Learn how to face given second-order linear homogeneous differential equation with constant coefficients.
b. Use different methods to familiarize with the solution, in case of linear inhomogeneous differential equations.
3. Numerical methods:
a. Use Euler, Taylor, and Runge-Kutta methods to get the approximate solutions of first-order differential equations.
b. Use up to date specialized mathematical software to get an approximate solutions of single differential equations or systems of differential equations.
c. Determine the difficulty of a problem based on the solution.

One of the great advantages of this subject is that it has a lot of interdisciplinary applications. Students from engineering courses are more motivated when they find the connection between topics from different subjects. As an example, authors from (Marchand & McDevitt) use the differential equations course to explore the mechanical vibrations of buildings during an earthquake.

3 Case Study: Project Proposal

Each learner group (or single student) must prepare a project that includes a paper and a presentation. The topic of the proposal and its development in such an assignment must be an engineering application concerning differential equations.

This case study is about the second-order ordinary differential equation that models a population evolution. In this case, students propose a problem, related to some ecosystems that can support a maximum population of P_M individuals of a certain species. The ecosystem, initially, it has a population of $N_0 = 1/5\ P_M$, while the growth rate is $1/20\ P_M$ individuals per month. At this moment an “explosion of population” takes place that affects the environmental balance so that the population exceeds P_M. Since the ecosystem does not support such situations there is a population decline that finally end up with declining oscillations around the value P_M.

The aim of the problem is to calculate the population, $N(t)$, and to find the maximum value reaches by $N(t)$ together with the point in time when it occurs.

Sometimes an *ad hoc* software development process is the appropriate. In fact, if the proposed report is small or noncritical, the use of a formal methodology may not be justified. The first challenge is to understand and define the work to be carried out.

To face the proposed problem, the student suggests several distinct stages:

To define the requirements.

1. To analyze and select between the different tools, the suitable one to solve the proposed situation.
2. Implementation with Matlab or Mathematica.
3. To look for other applications for the same problem.

3.1 Requirements

Every problem of numerical method usually has the same requirements: the solution we are looking for must be close to the exact one. Another requirement should be that the software development is as much efficient as possible with a good error processing.

With such short problem, the organized set of techniques that students considered as a method is targeted to achieve the objective, which means, in this case, the solution of the real problem. Wijers (Wijers, 1991) realized that, often, a method is focused on specifications practically feasible instead on how to carry out those specifications.

In the proposed projects the requirements where:

1. To find a suitable software to get the solution of the problem.
2. To develop an efficient programme to solve second-order ordinary differential equations (sometime this is already implemented in the mathematical software).
3. Test the program to be sure that it works properly for that kind of problems.

3.2 Find the Model of the Physical Situation

Numerous applications in engineering can be found related to and modelled by second-order linear differential equations, like the vibration of springs or electric circuits and modes ecological, biochemical, and compartmental systems in biology.

An “explosion” as the one mentioned in this case study, can be modelled by the equation:

$$\frac{d^2N}{dt^2}+\frac{dN}{dt}+\frac{5}{4}N=\frac{5}{4}P_M \qquad (1)$$

where N is the number of individuals, t is time in months, and P_M the maximum population of individuals as was indicated at the problem formulation.

A second-order linear differential equation has the general form:

$$p_1(x)\frac{d^2y}{dx^2}+p_2(x)\frac{dy}{dx}+p_3(x)y=f(x) \qquad (2)$$

where p_1, p_2 and p_3 are continuous functions in x. The case where $f(x) = 0$ is called homogeneous linear equations. When $f(x)\neq 0$ (for some x), like Equation (2), is called non-homogeneous.

In case of second-order Equation (2), an initial value problem (IVP) is to find a solution to the differential equation that satisfies some conditions, called initial conditions like:

$$y(x_0)=y_0 \qquad y'(x_0)=y_1$$

considering y0 and y1 given constants. If $p1$, $p2$, $p3$ and f in Equation (2) are differentiable on an

interval and $p_1(x)\neq 0$ there, then the Picard theorem (Mathews & Fink, 2004) -that was previously explained in classes as part of the course syllabus- ensures that the solution exists and it is unique to this IVP.

With the Equation (1), the following initial conditions are considered :

y(0)=1/5 y’(0)=1/20

at this point ,students try to find numerical handwritten solution to be sure that they find the right software that’s leads to correct solution. This is what they found

$$N(t)=P_M\left(1-\left(\frac{4}{5}\cos(t)+\frac{7}{20}\sin(t)\right)e^{-\frac{1}{2}t}\right)$$

Maximum value is $t = 3.09$ months, with $N(3.09) = 1.167\,P_M$.

3.3 Selection of the Suitable Tool

We have considered a computer-based application as a suitable tool that supports the use of a modelling technique. Such a tool includes modelling functionalities that allow the object transformation from real systems into models, checking the possible models to get the most adequate and consistent, changing results from one model and representation to another if it is needed and providing specifications to review previous results (Olle, Sol, & MacDonald).

Most of the time students choose Matlab as the tool to carry out the project, because a campus license provides Matlab and add-on tools for academic research and curriculum development at the University of Salamanca.

The students that go forward with the project mentioned in this paper have used Matlab during their Automation course and they realized that a block design is useful to test the solution of the problem (see Figure 1) and they also get a graphical representation of the solution (see Figure 2)

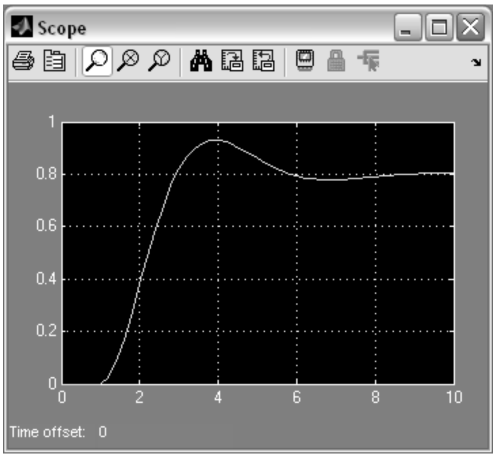

Figure 1: Block design with Matlab Simulink.

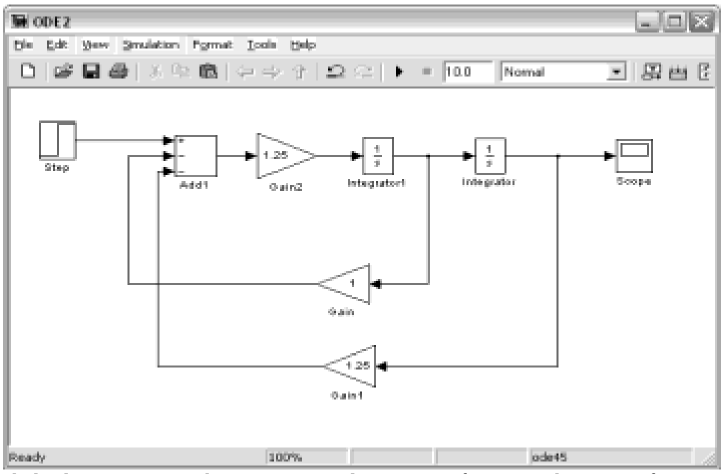

Figure 2: The Scope block displays its input with respect to simulation time. If we open the Scope after a simulation, the graphical representation of the solution will be displayed

3.4 Implementation using Matlab

To obtain symbolic solutions to ordinary differential equations (ODE), some students try to define and implement a function, but it is easy to find the Matlab command dsolve that computes easily and faster the solution sought (Moler & Costa, 1997). If dsolve command is used, to denote differentiation inside the equations, we use symbolic expressions containing the letter D. In that situation, D2, D3, ... DN, correspond with the second, third, ..., and Nth derivative, respectively. Thus, D2y is the term corresponding to d^2y/dt^2. In those expressions, the dependent variables are preceded by D and the independent variable is *t*. The independent variable can be changed to a different symbolic variable if we include that variable as the last input argument. Initial conditions, as always, can be specified by additional equations. If we considered an ODE without initial conditions, the solution contains constants of integration, A, B, etc.

3.5 The Assessment Process

For The evaluation is not only the final act of the instruction process or as mere mechanism of classification and selection of the students. The assessment method is considered as the basic platform for the direction of the learning process of the students and their self-training (autonomous learning). It allows us to learn from our own practice and to improve our system of education, that is to say, investigation in the teaching action.

The course evaluation, broadly speaking can be characterized as the set of activities that lead to emit a judgment on a student based on criteria previously established in the subject.

It is necessary to notice that we consider the assessment as a part of the teaching-learning process, not something external or addition artificially to this process.

4 Conclusions

The students developing any term project should question the knowledge and competences acquired during the course and look at it from a different perspective. As future engineers, it is expected that students work deeper to understand the situations and problems that they will find in their daily work. In order to do so, it is necessary to develop certain skills and competences, and remember learned issues during their studies in the research field.

This paper outlines the competences, knowledge and tools that an engineer must put into practice in a specific field. This study has analyzed one specific problem: a second-order ordinary differential equation that models a population evolutio

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