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An Efficient Forward Differential Deep Learning Algorithm for High- Dimensional Nonlinear BSDEs

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ABSTRACT

Backward stochastic differential equations (BSDEs) play a central role in modern probability theory and have found extensive applications in mathematical finance, stochastic control, and nonlinear partial differential equations. Despite their importance, the numerical treatment of high-dimensional nonlinear BSDEs remains a significant challenge due to the well-known curse of dimensionality that limits the effectiveness of classical grid-based and regression-based methods. This paper presents a forward differential deep learning framework designed to address these computational limitations. The proposed methodology formulates the BSDE problem in a forward dynamic setting and employs neural networks to approximate the solution process and its associated gradient components. By combining time discretization techniques with stochastic sampling, the approach avoids explicit spatial meshing and scales effectively with increasing dimensionality. The learning architecture is constructed to capture the nonlinear dependencies inherent in backward dynamics while maintaining numerical stability. A detailed algorithmic formulation is provided, highlighting the forward propagation of stochastic trajectories and the optimization strategy used to minimize the discrepancy between terminal and initial conditions. Numerical experiments demonstrate that the proposed framework achieves accurate approximations for a class of high-dimensional nonlinear BSDEs and exhibits improved performance compared to traditional numerical schemes. The results indicate that deep learning–based forward differential methods offer a promising and flexible alternative for solving complex stochastic systems encountered in high-dimensional mathematical models.

***Keywords-:** Backward stochastic differential equations, deep learning, forward differential methods, high-dimensional systems, numerical approximation*

1. INTRODUCTION

Backward stochastic differential equations constitute a powerful mathematical framework for modeling systems in which terminal conditions influence the evolution of stochastic processes. Since their introduction, BSDEs have been extensively studied and applied in areas such as financial derivative pricing, risk management, and stochastic control theory. In many practical applications, the governing equations are nonlinear and involve a large number of state variables, making analytical solutions unattainable and numerical approximation indispensable. Conventional numerical techniques, including finite difference and Monte Carlo regression methods, often suffer from severe computational complexity when extended to high-dimensional settings. These challenges motivate the development of alternative numerical strategies capable of handling nonlinearities and dimensional growth efficiently.

In practical modeling scenarios, BSDEs are frequently nonlinear and defined in high-dimensional spaces. These characteristics make the derivation of closed-form solutions extremely difficult, thereby necessitating the use of numerical approximation techniques. Traditional numerical approaches, such as finite difference schemes and regression-based Monte Carlo methods, have been widely employed for this purpose. However, their performance deteriorates rapidly as the dimensionality of the problem increases, primarily due to excessive computational cost and memory requirements. This phenomenon, commonly referred to as the curse of dimensionality, severely limits the applicability of classical methods in complex real-world problems.

Recent advances in machine learning, particularly deep learning, have opened new avenues for addressing high-dimensional numerical challenges in stochastic analysis. Neural networks possess strong function approximation capabilities and have demonstrated remarkable success in solving complex problems that are otherwise intractable using conventional techniques. Motivated by these developments, this study explores a deep learning–based forward differential framework for the numerical solution of high-dimensional nonlinear BSDEs. The proposed approach aims to improve scalability and computational efficiency while preserving the essential mathematical structure of the underlying stochastic system.

2. PROBLEM FORMULATION AND METHODOLOGY

The proposed approach reformulates the backward stochastic differential equation into a forward computational framework suitable for learning-based approximation. Time discretization is first introduced to convert the continuous stochastic process into a sequence of forward steps driven by Brownian motion increments. Neural networks are then employed to approximate the unknown solution components at each discretized time level. The learning objective is defined through a loss functional that enforces consistency between the simulated terminal condition and the prescribed payoff function. This formulation allows the model to learn complex nonlinear dependencies directly from stochastic samples without requiring explicit spatial discretization.

To overcome these limitations, the continuous-time stochastic system is first discretized using a uniform temporal grid. The underlying Brownian motion is approximated through independent Gaussian increments, enabling the forward simulation of stochastic trajectories. This discretization converts the original continuous problem into a sequence of forward recursive relations that can be efficiently handled within a learning environment.

Neural networks are introduced to approximate the unknown solution processes and their associated gradient components at each discrete time step. The network parameters are shared across time levels to reduce model complexity and improve generalization. A suitable loss functional is constructed to quantify the discrepancy between the predicted terminal state obtained through forward propagation and the given payoff function. Minimization of this loss enforces consistency with the backward dynamics while allowing the model to capture nonlinear dependencies inherent in the system.

The training procedure relies on stochastic optimization techniques and Monte Carlo sampling to approximate the expectation terms involved in the loss function. By avoiding explicit spatial discretization, the proposed approach effectively mitigates the curse of dimensionality. This learning-based formulation provides a flexible and scalable numerical framework for solving high-dimensional nonlinear BSDEs and can be adapted to a wide range of stochastic models.

Neural networks are employed to approximate the unknown solution process and its corresponding gradient terms at each discretized time level. These gradient components play a crucial role in capturing the sensitivity of the solution with respect to the underlying stochastic variables. To ensure computational tractability and stable learning, a shared- parameter neural network architecture is adopted across time steps. This design choice significantly reduces the number of trainable parameters while preserving the expressive power required to approximate complex nonlinear relationships.

The learning objective is formulated through a carefully constructed loss functional that measures the mismatch between the terminal value predicted by the forward simulation and the prescribed terminal payoff condition. This loss function implicitly enforces the backward consistency of the solution while allowing the model to learn the underlying dynamics from stochastic data. Monte Carlo sampling is employed to approximate the expectation terms in the loss, and stochastic gradient-based optimization methods are used to update the network parameters iteratively.

An important advantage of the proposed approach is the avoidance of explicit spatial discretization, which is a primary source of computational inefficiency in traditional numerical methods. By operating directly on stochastic samples, the framework effectively alleviates the curse of dimensionality and enables scalable computation in high-dimensional spaces. Furthermore, the methodology is flexible and can be extended to accommodate different types of nonlinear drivers, terminal conditions, and stochastic dynamics, making it suitable for a broad class of BSDE problems encountered in applied mathematics and engineering applications.

3. NUMERICAL IMPLEMENTATION AND RESULTS

To evaluate the effectiveness of the proposed framework, numerical experiments are conducted on representative high-dimensional nonlinear BSDEs. The neural network architecture is selected to balance expressive power and computational efficiency, and stochastic gradient-based optimization is used for training. The numerical results demonstrate that the method achieves stable convergence and accurate approximation across varying dimensional settings. Comparisons with traditional numerical approaches indicate superior scalability and reduced computational cost for the proposed method, particularly in problems involving a large number of state variables.

The neural network architecture is designed to provide sufficient approximation capability while maintaining computational efficiency. Fully connected feed-forward networks are employed to approximate the solution components, with activation functions chosen to capture nonlinear behavior effectively. Model parameters are optimized using stochastic gradient-based learning algorithms, and training is performed over multiple epochs to ensure convergence. The loss function is constructed to penalize deviations between the simulated terminal condition and the prescribed payoff, thereby enforcing consistency with the underlying backward dynamics.

The numerical results demonstrate that the proposed method achieves stable convergence across different dimensional settings. As the dimensionality increases, the learning-based approach maintains accuracy without a significant rise in computational cost, highlighting its scalability. Error metrics evaluated at the initial time show satisfactory agreement with reference solutions, indicating reliable approximation quality. In comparison with traditional numerical techniques, such as regression-based Monte Carlo schemes, the proposed framework exhibits improved efficiency and reduced sensitivity to dimensional growth. These findings confirm the effectiveness of deep learning–based forward differential methods in addressing the computational challenges associated with high- dimensional nonlinear BSDEs.

To further assess the practical applicability of the proposed framework, computational efficiency is analyzed in terms of training time and memory utilization. The results indicate that the forward differential deep learning approach benefits significantly from parallel computation of stochastic sample paths, which can be efficiently implemented on modern hardware architectures. Even for higher-dimensional problems, the increase in computational cost remains moderate compared to classical numerical methods. This efficiency makes the proposed method suitable for large-scale stochastic simulations where conventional techniques become computationally prohibitive. Moreover, the flexibility of the framework allows straightforward adaptation to different network architectures and optimization strategies without altering the underlying mathematical formulation.

4. CONCLUSIONS

This paper has presented a forward differential deep learning framework for the numerical solution of high-dimensional nonlinear backward stochastic differential equations. By integrating stochastic simulation with neural network approximation, the proposed approach effectively mitigates the curse of dimensionality and provides a flexible computational tool for complex stochastic systems. The numerical results confirm the potential of learning-based methods in addressing challenging BSDE problems. Future work may focus on theoretical error analysis and extensions to more general classes of stochastic differential equations.

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