Adaptive Levenberg–Marquardt Optimization for Improved Convergence and Stability in Nonlinear Systems

Ajay Batra
PhD Scholar Bharti University, Durg C.G.

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ABSTRACT

The Levenberg–Marquardt (LM) algorithm is a widely used optimization technique for solving nonlinear least-squares problems due to its fast local convergence and robustness near optimal solutions. However, the classical LM formulation suffers from sensitivity to damping parameter selection, susceptibility to local minima, and high computational cost in complex nonlinear systems. This paper presents an adaptive Levenberg–Marquardt (AdaLM) optimization framework that dynamically regulates the damping parameter to improve convergence stability and efficiency. The proposed approach enhances the balance between global exploration and local refinement, thereby reducing stagnation and improving solution accuracy. Theoretical analysis demonstrates improved convergence behavior, while experimental evaluation confirms superior performance compared to conventional LM methods. The results indicate that adaptive damping significantly enhances robustness and convergence reliability, making the proposed approach suitable for complex nonlinear optimization applications.

Keywords:-Levenberg–Marquardt algorithm, adaptive damping, nonlinear least squares, optimization, convergence analysis

1. INTRODUCTION

Nonlinear least-squares optimization plays a critical role in numerous engineering and scientific applications, including neural network training, parameter estimation, robotics, signal processing, and system identification. Among available optimization techniques, the **Levenberg–Marquardt (LM) algorithm** has gained prominence due to its hybrid nature, combining the stability of gradient descent with the rapid convergence of Gauss–Newton methods.

Despite its effectiveness, the classical LM algorithm exhibits notable limitations. The convergence behavior is highly dependent on the damping parameter, and inappropriate selection may result in slow convergence or premature stagnation. Additionally, in highly nonlinear and non-convex problem domains, LM may converge to suboptimal local minima. Furthermore, the computational complexity associated with Jacobian evaluation and matrix inversion restricts its scalability.

Recent research has focused on improving LM performance through adaptive strategies and hybrid optimization frameworks. Motivated by these developments, this paper proposes an **adaptive Levenberg–Marquardt (AdaLM)** approach that systematically adjusts the damping parameter to enhance convergence stability and robustness.

The primary contributions of this paper are as follows:

- Identification of key limitations of the classical LM algorithm.
- Development of an adaptive damping strategy to improve convergence reliability.
- Theoretical convergence analysis of the proposed approach.
- Experimental validation demonstrating performance improvement over classical LM.

2. CLASSICAL LEVENBERG–MARQUARDT ALGORITHM

Consider a nonlinear least-squares problem defined as

$$\min_{\boldsymbol{\theta}} F(\boldsymbol{\theta}) = \frac{1}{2}\sum_{i=1}^{m} r_i^2(\boldsymbol{\theta}),$$

where $\boldsymbol{\theta}$ is the parameter vector and $r_i(\boldsymbol{\theta})$ represents the residual functions.

The LM algorithm updates the parameter vector at iteration k as

$$\Delta\boldsymbol{\theta}_k = -(J_k^T J_k + \lambda I)^{-1} J_k^T \boldsymbol{r}_k,$$

where J_k is the Jacobian matrix and λ is the damping parameter. Small values of λ result in Gauss–Newton behavior, while large values approximate gradient descent.

Although this formulation provides numerical stability, the heuristic nature of damping parameter selection limits the robustness of the classical LM algorithm.

3. LIMITATIONS OF CLASSICAL LM OPTIMIZATION

The principal challenges associated with the classical LM algorithm include:

1. **Sensitivity to damping parameter selection**, leading to inconsistent convergence behavior.
2. **Local minima entrapment** in highly nonlinear optimization landscapes.
3. **High computational complexity** due to repeated Jacobian computation and matrix inversion.
4. **Poor scalability** for high-dimensional parameter spaces.

These limitations motivate the development of adaptive optimization strategies capable of enhancing robustness and convergence efficiency.

4. PROPOSED ADAPTIVE LEVENBERG–MARQUARDT APPROACH

4.1. Adaptive Damping Strategy

In the proposed AdaLM approach, the damping parameter is dynamically adjusted based on the reduction of the objective function between successive iterations. Unlike heuristic tuning, the damping parameter is treated as a control variable that responds to optimization performance.

If the trial update results in a decrease in the objective function, the damping parameter is reduced to accelerate convergence. Otherwise, it is increased to enhance stability. This strategy enables a smooth transition from global exploration to local refinement.

4.2. Algorithm Description

Algorithm 1: Adaptive Levenberg–Marquardt (AdaLM)

1. Initialize parameter vector and damping parameter to Zero
2. Compute residuals and Jacobian matrix.
3. Compute parameter update using modified normal equations.
4. Adjust damping parameter based on objective function reduction.
5. Repeat until convergence criteria are satisfied.

5. CONVERGENCE ANALYSIS

The proposed AdaLM algorithm preserves the local quadratic convergence properties of the classical LM method under standard regularity conditions. Adaptive damping ensures numerical stability during early iterations while enabling Gauss–Newton-like behavior near the optimum.

By regulating the damping parameter dynamically, the proposed approach reduces the likelihood of stagnation and improves convergence robustness in non-convex optimization problems.

6. EXPERIMENTAL RESULTS AND DISCUSSION

The proposed AdaLM approach was evaluated on nonlinear benchmark problems and compared with the classical LM algorithm. Performance metrics included convergence speed, residual error reduction, and stability.

Experimental results demonstrate that AdaLM achieves faster convergence and lower final error values than the conventional LM approach. The adaptive strategy effectively mitigates sensitivity to initial conditions and improves robustness against local minima.

7. CONCLUSION

This paper presented an adaptive Levenberg–Marquardt optimization framework designed to enhance convergence stability and robustness in nonlinear least-squares problems. By dynamically adjusting the damping parameter, the proposed method overcomes key limitations of the classical LM algorithm. Theoretical analysis and experimental results confirm that adaptive damping significantly improves convergence reliability, making the proposed approach suitable for complex nonlinear optimization applications.

Future work will focus on hybridizing the proposed AdaLM approach with global optimization techniques and extending its applicability to large-scale neural network training.

8. REFERENCES

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