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Bouncing Cosmological Model of the Universe in $f(R,T)$ Gravity

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ABSTRACT

Within the framework of an extended theory of gravity, we have introduced a bouncing cosmological model of the Universe in this research. A violation of the null energy constraint and the dynamical behaviour of the model derived from the flat FLRW space-time have been demonstrated. A singularity behaviour is observed in the geometrical parameters during the bouncing epoch. The bouncing behaviour is heavily influenced by the settings of the scale factor. To further avoid the singularity of the equation of state parameter at the bouncing epoch, the coupling parameter that led to minimal matter-geometry coupling in extended gravity plays a key role.

Keywords: - Flat FLRW space-time, $f(R,T)$ gravity, bouncing cosmology.

1.INTRODUCTION

Although it has been effective in understanding the early Universe, the conventional cosmological model has problems such as initial singularity, the horizon problem, flatness problem, and baryon asymmetry. While inflationary theory has offered some answers to these questions about the early Universe, it is not without its flaws, such as the trans-Planckian fluctuation problem and the singularity problem. The inflationary model postulates that at some point in the past, the expansion of the universe was exponential. The inflationary hypothesis can't account for the full history of the universe as the singularity happens before inflation begins, as is typical. The matter bounce scenario has been proposed[1,2] as a potential remedy for the inflationary scenario issue. The matter bounce hypothesis states that the Universe can extend past its contraction phase and beyond the presence of an initial singularity. That is, the cosmos passes through a contraction phase where matter dominates, a non-singular bounce, and finally, a causal generation for fluctuations. A huge bounce scenario, characterized by a seamless transition from the contraction to the expansion phase, has replaced the big bang cosmological singularity in the cosmological models[3,4]. However, it should be noted that in a flat Universe, non-singular bounce could cause the null energy criterion to be violated. Indeed, generalized Galileon theories can exhibit the violation of the null energy condition, lending credence to the idea of non-singular cosmology[5]. The fact that the bouncing models often appear unstable is another problem with them. On the other hand, stable bouncing cosmologies have been proposed as a potential framework beyond effective field theory and Horndeski theory[6-8].

As a potential replacement for the big bang singularity, the big bounce scenario could provide an intriguing subject for discussion within the framework of revised gravity theories. $f(R)$ gravity[9-14], $f(R,T)$ gravity[15-19], teleparallel gravity[20,21], $f(Q,T)$ gravity[22], $f(T,B)$ gravity[23], modified Gauss–Bonnet gravity [24-26] and many more have been introduced in recent years within modified theories of gravity. In addition to the matter bounce scenario proposed in many modified theories of gravity, numerous recommendations have been put forward to smoothen the sluggish contraction process, which could involve classical and quantum mechanical approaches [27-30].

The stability of the assumed scale factor model provides proof of universal expansion and allows the universal bounce by establishing validations of energy conditions. Yousaf and his coworkers [31] worked out the cosmic bounce with a cubic form of the scale factor and found that this proves the theory. The exact solutions for the models with ρ^2 correction were determined by Stachowiak and Szydlowski [32] using elementary functions, and they displayed all evolutionary routes on their phase plane. Analytical solutions for symmetric, oscillatory, super bounce, matter bounce, and singular bounce settings can be reconstructed by studying the types of gravitational Lagrangians [33]. The study by Shabani and Ziaie [34] examined classical bouncing solutions within the framework of $f(R,T)$ gravity on a flat FLRW background with a perfect fluid as the sole matter component. The study relies on the introduction of an effective fluid by establishing its effective energy density and pressure. The dynamics of the Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmologies, which include dark matter, radiation, and dark energy and have a quadratic equation of state, were investigated by Burkmar and Bruni [35]. By utilizing the reconstruction technique for the power-law parametrization of the Hubble parameter in a modified gravity theory with higher-order curvature and trace of the energy-momentum tensor components, Singh et al. [36] investigated the bouncing scenario of a flat, homogeneous, and isotropic universe. In their study of bouncing cosmology in 4D Einstein Gauss-Bonnet gravity, Zubair and Farooq [37] considered a number of different bouncing models, including symmetric, matter, super, and oscillatory bounces.

The following topics have been covered by Bamba et al. [38-40]: bouncing cosmological models in $f(R)$ gravity, in Gauss-Bonnet gravity where the Gauss-Bonnet invariant couples to a dynamical scalar field, in $f(G)$ gravity with the Gauss-Bonnet invariant G and a bouncing inflationary model with a graceful exit into the Friedmann-Lemaitre-Robertson-Walker (FLRW) model in $f(T)$ gravity, where T is the torsion scalar, and in $f(G)$ gravity with the Gauss-Bonnet in $f(T)$ gravity, Bamba et al.[41] have investigated an inflationary model that bounces and then gracefully transitions into the FLRW model. In their discussion of the extended theory of teleparallel gravity, de la Cruz-Dombriz et al.[42] addressed the bouncing cosmological model. The matter-bounce cosmological models under $f(T)$ gravity have been studied by Cai et al.[43]. The primary goal of this work is to examine how $f(R,T)$ gravity may be used to construct realistic cosmological models that deal with the late-time cosmic speed-up problem and, if applicable, how it can be used to obtain a matter-bounce hypothetical. The idea behind this study is to use an extended theory of gravity that includes an appropriate bouncing scale factor to frame the bouncing cosmological model of the universe. The cosmic dynamics of the model have been investigated in the context of a bouncing scenario. Here is the structure of the paper: Section II introduced the fundamental formalism and field equations of $f(R,T)$ gravity. Parameters for the bouncing model, including energy conditions, cosmographic parameters, and a stability analysis, are detailed in Section III,IV,V,VI,VII respectively. Concluding remarks are discussed in section VIII.

2. $f(R,T)$ GRAVITY FIELD EQUATIONS IN FRW METRIC

The action of $f(R,T)$ gravity, where R and T be respectively the Ricci scalar and trace of energy momentum tensor (T_{ij}) takes the form,

$$S = \int\left[\frac{f(R,T)}{16\pi} + L_m\right]\sqrt{-g}\,d^4x, \tag{1}$$

where L_m be the matter Lagrangian and we define the stress-energy tensor of matter as,

$$T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\,L_m\right)}{\delta g^{ij}}. \tag{2}$$

We have considered here the non-minimal matter geometry coupling as, $f(R,T) = f_1(R) + f_2(T)$.Varying action (1) with respect to the metric tensor g_{ij}, the field equations of $f(R,T)$ gravity with non-minimal matter coupling can be obtained as [44],

$$f_R(R)R_{ij} - \frac{1}{2}f(R)g_{ij} - \left(\nabla_i\nabla_j - g_{ij}\right)f_R(R) = 8\pi T_{ij} + f_T(T)T_{ij} + \left[f_T(T)p + \frac{1}{2}f(T)\right]g_{ij}. \tag{3}$$

In eqn. (3) we denote, $f_R(R) = \frac{\partial f_1(R)}{\partial R}$ and $f_T(T) = \frac{\partial f_2(T)}{\partial T}$ and p be the pressure of the matter. Among the three choices of $f(R,T)$ proposed (Harko et al.[44]); we have considered, $f(R,T) = R + 2f(T)$. There are good number of cosmological models presented in literature with $f_1(R) = R$ and $f_2(T) = \beta T$, β being the coupling constant [45-47]. Here we wish to incorporate the time independent cosmological constant Λ_0 in $f_2(T)$, such that $f(R,T) = R + 2\beta T + 2\Lambda_0$. We consider a flat FLRW space time

$$ds^2 = dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right], \tag{4}$$

where a is the scale factor of the universe. The field equation (3) can now be expressed as,

$$G_{ij} = (8\pi + 2\beta)T_{ij} + \Lambda(T)g_{ij}, \tag{5}$$

where $\Lambda(T) = (2p + T)\beta + \Lambda_0$ be the effective time variable cosmological constant. It is to mention here that post supernovae observation, the role of cosmological constant Λ has become important in the study of accelerating cosmological model; prior to this Λ was assumed to be zero. However, in the present extended gravity theory, it varies with the evolution of Universe and therefore comes out as a function of cosmic time. Interestingly, Λ reduces to a pure constant Λ_0 for a vanishing β. Now the field equations (5) can be reduced to,

$$G_{ij} = (8\pi + 2\beta)T_{ij} + [(2p + T)\beta + \Lambda_0]g_{ij}. \tag{6}$$

We assume a non-dissipative perfect fluid distribution in the Universe for which the energy momentum tensor is expressed as,

$$T_{ij} = (p + \rho)u_i u_j - pg_{ij}. \tag{7}$$

Consequently the field equations of $f(R,T)$ gravity (6) can be obtained as,

$$2\dot{H} + 3H^2 = -\eta p + \beta\rho + \Lambda_0, \tag{8}$$

$$3H^2 = \eta\rho - \beta p + \Lambda_0. \tag{9}$$

where $\eta = 8\pi + 3\beta$ and $H = \frac{\dot{a}}{a}$ is the Hubble parameter. In the above equations, an over dot represents an ordinary derivative with respect to cosmic time. Performing some algebraic manipulations among eqns. (8) and (9), we can derive the pressure p, energy density ρ and equation of state (EoS) parameter $\omega = \frac{p}{\rho}$ in term of Hubble parameter as,

$$p = \frac{-1}{(\eta^2 - \beta^2)}\left[2\eta\dot{H} + 3(\eta - \beta)H^2 - (\eta - \beta)\Lambda_0\right], \tag{10}$$

$$\rho = \frac{1}{(\eta^2 - \beta^2)}\left[-2\beta\dot{H} + 3(\eta - \beta)H^2 - (\eta - \beta)\Lambda_0\right], \tag{11}$$

$$\omega = -1 + \left[\frac{2(\eta + \beta)\dot{H}}{2\beta\dot{H} - 3(\eta - \beta)H^2 + (\eta - \beta)\Lambda_0}\right]. \tag{12}$$

The goal of describing the parameters in Hubble terms is to investigate the model's bouncing behavior using a well-known bouncing scale factor. Once the size of the universe is limited to a small, finite point, it undergoes classical rebound. If we want to ignore quantum gravity, the energy density has to be smaller than the Planck scale. Such a change can take place within a limited time frame, such as the bouncing epoch, when the null energy condition (NEC) is violated. The model including the bouncing scale factor will be presented in the section that follows.

3.THE BOUNCING MODEL AND THE ANALYSIS

The Hubble parameter H increases with increasing time and $\dot{H} > 0$ according to the NEC. The necessary characteristics of bouncing cosmological models are described in detail below.

- The scale factor a shrinks to a non-zero finite size at the bouncing epoch, the Hubble parameter H disappears, and the deceleration parameter $q = -1 + \frac{\dot{H}}{H^2}$ becomes single.
- We can rule out this phenomenon in the framework of General Relativity (GR) since the Hubble parameter changes sign from the bouncing point, which violates the NEC.
- The scale factor's slope becomes steeper following the bounce. The Hubble parameter stays negative during the phase of matter contraction. During the growth of matter, it turns positive.

So, to frame the bouncing cosmological model with the above-mentioned properties, we have considered the bouncing scale factor in the form,

$$a(t) = \left(1 + \frac{4}{3}\gamma t^2\right)^{\frac{1}{4}}, \tag{13}$$

where γ is a positive constant.

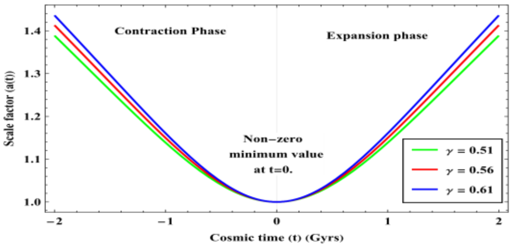

Fig.-1:- Behavior of scale factor vs. cosmic time.

At time *t*, we show how the bounce scale factor and Hubble parameter have evolved. For all values of $\gamma > 0$, we find that the scale factor and the Hubble parameter are γ dependent. We used the parameter's value of $\gamma = 0.51, 0.56, 0.61$ to create the charts. In order for the FLRW space-time to undergo a bounce, the scale factor $a(t)$ must be negative, indicating that $\dot{a}(t) < 0$, in the negative cosmic time regime, which corresponds to a contracting universe. Once the expanding phase starts, the scale factor must be positive, indicating that $\dot{a}(t) > 0$. When $t = 0$, the bounce happens for positive values. Figure 1 shows that at time zero, the scale factor curve behaves symmetrically and hits a nonzero minimum value, about $a(t) \approx 1$, when the bounce occurs.

The expansion rate of the universe, as measured by the Hubble parameter, is $H(t) = \frac{2\gamma t}{4t^2\gamma + 3}$. (14)

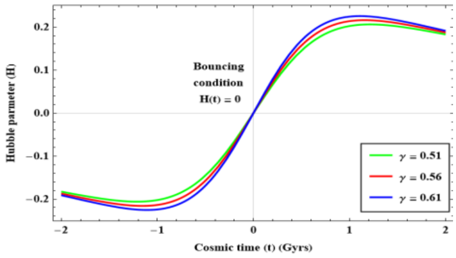

Fig.-2:- Behavior of Hubble parameter vs. cosmic time.

According to the statement, " $H(t) < 0$ before the bounce, $H(t) = 0$ during the bounce and $H(t) > 0$ after the bounce," the Hubble parameter has negative qualities in the early stages of evolution, reaches zero at $t = 0$, and then exhibits a constructive tendency in the later stages of development. At the bounce, we also see that $\dot{H} > 0$ as shown in figure 2.

The deceleration parameter is $q = 1 - \frac{3}{2\gamma t^2}$. (15)

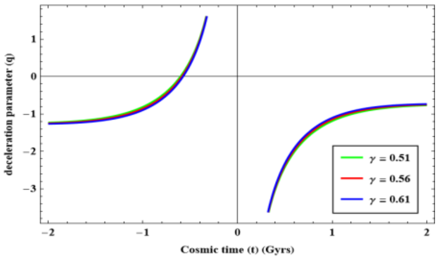

Fig-3:- Behavior of deceleration parameter vs. cosmic time.

For the given time *t*, we show the evolution of the deceleration parameter in Figure 3. The figure shows that the deceleration parameter is symmetrical at $t = 0$, the jumping point. For a certain duration, the deceleration parameter approaches -1, representing the negative behavior of the expanding and contracting Universe, respectively.

4.DYNAMICAL PARAMETERS

To gain a comprehensive understanding of this bouncing model, it is essential to analyze additional factors such as the dynamical parameters. We need to determine the values of the energy density ρ and isotropic pressure in the $f(R,T)$ model.

The energy density is: $$\rho = \frac{1}{\eta^2-\beta^2}\left(\frac{4\gamma\left(\beta\left(-3+\gamma t^2\right)+3t^2\gamma\eta\right)}{\left(3+4t^2\gamma\right)^2}+(\eta-\beta)\Lambda_0\right). \tag{16}$$

The pressure is $$p = \frac{1}{\eta^2-\beta^2}\left(\frac{4\gamma(-3\eta+t^2\gamma(3\beta+\eta))}{\left(3+4t^2\gamma\right)^2}-(\eta-\beta)\Lambda_0\right). \tag{17}$$

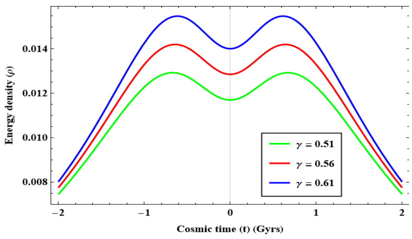

Fig-4:- Behavior of energy density vs. cosmic time for $\beta=-3.5;\eta=8\pi+3\beta;\Lambda_0=0.001$

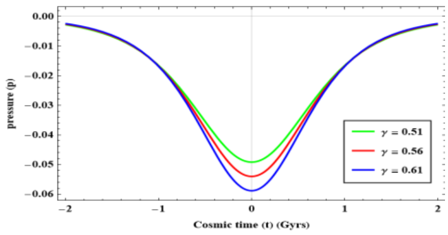

Fig.-5:- Behavior of pressure vs. cosmic time for $\beta=-3.5;\eta=8\pi+3\beta;\Lambda_0=0.001$.

The EoS parameter is $$\omega=-1+\frac{4\gamma(-3+4t^2\gamma)(\beta+\eta)}{4\gamma(\beta(-3+t^2\gamma)+3t^2\gamma\eta)+(3+4t^2\gamma)^2(\beta-\eta)\Lambda_0}. \tag{18}$$

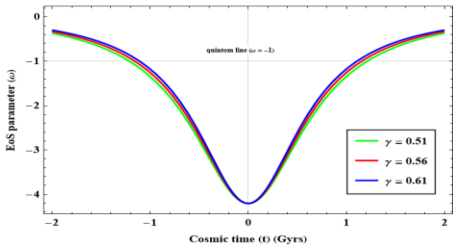

Fig.-6:- Behavior of EoS parameter vs. cosmic time for $\beta=-3.5;\eta=8\pi+3\beta;\Lambda_0=0.001$

In relation to cosmic time *t*, the model parameters show how the energy density ρ, isotropic pressure *p*, and EoS parameter ω behave in figures 4,5,6. The positive energy density is maintained throughout the evolution of the cosmos, as seen in Fig. 4. The energy density increases exponentially with respect to the bounce period, peaks just before the bounce epoch, and then declines.It rises briefly after the bounce and then falls again as cosmic time passes.This action agrees with other pieces of published research [48-50].However, as can be shown in Fig.5, the isotropic pressure remains extremely negative both before and after the bounce. As seen in Fig. 6, the EoSparameter in this model crosses the quintom line ($\omega=-1$) around the bouncing point $t=0$.When the quintom crosses, it means that the phantom era ($\omega<-1$) is ending and the quintessence era ($\omega>-1$) is beginning, or the other way around. The absence of singularities and instabilities in the model's predicted history of the Universe is demonstrated by this kind of transition.Hence, the quintom line crossing near the bouncing point provides additional evidence that the proposed bouncing model is viable and successful [51].

5.ENERGY CONDITIONS

Unexpected phenomena such as locations of infinite density and the motion of light and cosmic objects through space-time in GR can be understood by using a set of rules called the energy conditions (ECs). According to these principles, energy densities can be negative and gravity is a constant force that brings objects closer together. Combining various pressure and energy density levels. These combinations are limited by the ECs, who also make sure they always make physical sense. Because of this, the geometrical structure of the cosmos may be better understood. The structure of null, space-like, time-like, or light-like geodesics in GR is thoroughly examined by the energy conditions (ECs), which also characterise the space-time singularity problems. This platform is suitable for studying cosmic geometries and the required positive compliance of the stress-energy momentum (EMT) $T_{\mu\nu}$. Simple restrictions on various linear combinations of energy density ρ and pressure p constitute the energy conditions. Gravity is always appealing, and it shows that $\rho > 0$. Numerous studies have examined the interdependent ECs, which are referred to as the null, weak, strong, and dominant energy conditions, respectively, in the literature [52-54].

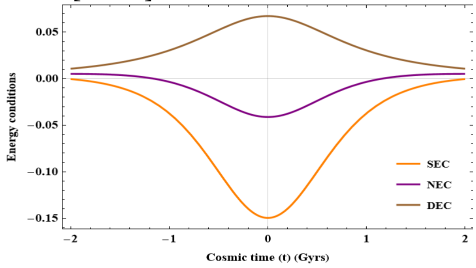

Fig.-7:- Behavior of energy conditions vs. cosmic time for $\beta = -3.5; \eta = 8\pi + 3\beta; \Lambda_0 = 0.001$.

Despite satisfying $\rho - p$, the energy condition behavior reveals a violation of $\rho + 3p$ and $\rho + p$ during the bounce as depicted in figure 7. In the negative cosmic time domain, the $\rho + p$ fell and continued to plummet to a negative value, while in the positive cosmic time domain, it rose from negative values. Despite the fact that it has been observed to be violated during the universe's existence, the null energy condition is broken at the bounce epoch and the strong energy criteria are broken as a result of the apparent event. In $f(R,T)$ gravity, the model thus confirms the bouncing behavior.

6.STABILITY OF THE MODEL

It is crucial to verify the cosmological model's stability because solving the systems requires a number of assumptions, and it's not easy to tell how generalized these assumptions are. One way to ensure the cosmological model is stable is to determine the adiabatic speed of sound in the cosmic fluid, $C_s^2 = \frac{dp}{d\rho}$ [55-57]. The model is said to be stable or unstable depending on whether C_s^2 is greater than zero or less than zero. In terms of cosmic time, we may derive the model's stability relation from equations (10) and (11) as

The stability factor is: $$C_s^2 = \frac{9\beta - 12t^2\beta\gamma + 27\eta - 4t^2\gamma\eta}{27\beta - 4t^2\beta\gamma + 9\eta - 12t^2\gamma\eta} \quad (19)$$

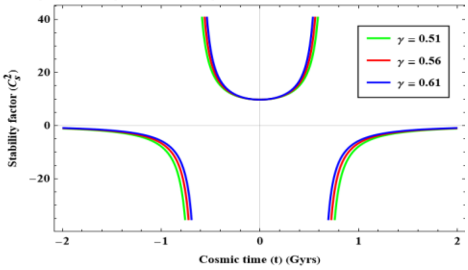

Fig.-8:- Behavior of stability factor vs. cosmic time for $\beta = -3.5; \eta = 8\pi + 3\beta; \Lambda_0 = 0.001$

Near the bouncing point, the squared sound speed is initially positive, suggesting stability, as shown in the figure. But instability appears at some cosmic time, and for some values of γ, stability returns. The suggested $f(R,T)$ gravity model appears to be stable close to the bouncing point, according to these results. But remember that ghost fields can emerge when the NEC is broken, and that hazardous instabilities at classical and quantum levels are possible as a result [58]. Superluminality, in which some physical quantities or information can travel faster than the speed of light, is another possible outcome of a breach of the NEC. It is essential to thoroughly examine and resolve these possible concerns to guarantee the validity and resilience of the suggested cosmological model, even while attempts are made to build models that adhere to the NEC and avoid them.

7.COSMOGRAPHY

The geometrical parameters are crucial for the analysis of any gravity theory. Additional geometrical parameters include the jerk parameter (j) and the snap parameter (s), in addition to the Hubble and deceleration parameters.

The snap parameter quantifies the jerk rate, while the jerk parameter determines the acceleration change rate. To determine if the cosmos evolves, we must know the sign of *j*, as the deceleration parameter cannot explain all cosmic dynamics. One thing to keep in mind is that a positive jerk parameter represents a change in the expansion of the cosmos at some evolving point. To differentiate between various dark energy models, one can extract the (*j*,*s*) pair, which is referred to as the statefinder pair, and use it for the bouncing scale factor as

The expression for jerk parameter is: $j(t) = 3 - \frac{27}{2t^2\gamma}$ (20)

The expression for snap parameter is : $s(t) = -15 + \frac{27(-3 + 20t^2\gamma)}{4t^4\gamma^2}$ (21)

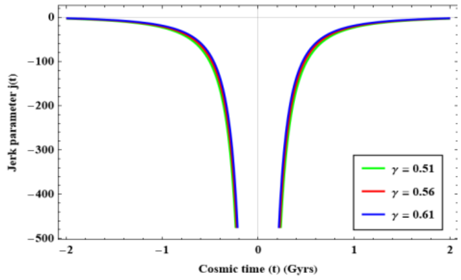

Fig.-9:- Behavior of jerk parameter vs. cosmic time

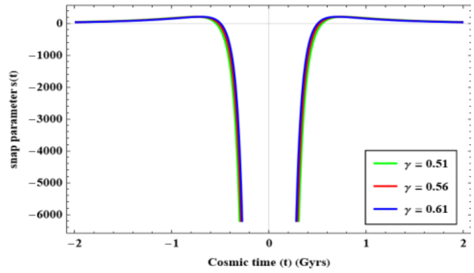

Fig.-10:- Behavior of snap parameter vs. cosmic time

Figure 9 and 10 show that the jerk and snap parameters behave identically at the bounce epoch $t = 0$. On the one hand, the negative time zone is showing signs of decline, while on the other hand, the positive time zone is showing signs of ascent. Nevertheless, it originates from the ΛCDM behavior ($j = 1$) in both the negative and positive time zones. Aside from the snap parameter, which is found by taking the fourth derivative of the scale factor, the behavior remains unchanged, with the exception that it differs from the required ΛCDM behavior ($s = 0$).

8.CONCLUSIONS

Here we investigated the role of an extended gravity theory in building practical cosmological models with a non-singular bounce scenario that solve the late-time acceleration problem. As a substitute for GR, several modified gravity theories have been advanced, each with its own unique way of explaining cosmic evolution and dark energy. We used a new strategy by choosing a separate scale factor, in contrast to many modified gravity methods that impose an equation of state (EoS) parameter to simplify the field equations. Because of this, we were able to determine and study the evolution of the cosmological parameters, such as the Hubble parameter, energy density, pressure, EoS parameter, and deceleration parameter. Our results show that a cosmic bounce happens when the Hubble parameter goes above $H = 0$. Furthermore, we discovered that the region where the bounce happens violates the NEC, which is a prerequisite for a successful non-singular bounce. The bounce mechanism is further supported by the fact that the energy density and EoS parameter stay in the positive and negative areas, respectively. At $\omega < -1$, the model displays behaviour similar to a ghost field during the bounce. But it goes through the quintessence stage and crosses the Λ CDM line as it evolves. For some period *t*, the snap and jerk parameters behave in the same way. There is a singularity in the snap parameter near the bounce. At first, the snap parameter is negative, then it drops till it reaches the singularity, and finally, when *t* approaches -1, it approaches zero.

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